Materials Processing: ME 7050
Assignment: Vector and Tensor Algebra

1. Find the angle between the two vectors from the origin to the two points A(1,2,1) and B(0,-3,2).

2. Find the area of the parallelogram formed on the two vectors of problem 1.

3. Find a unit vector perpendicular to both vectors of problem 1.

4. Write out (expand) the following expressions:

   (a) $t_i = T_{ji} n_j$

   (b) $e = \epsilon_{kk}$

   (c) $2W = T_{ij} \epsilon_{ij}$

5. New right handed coordinate axes are chosen with an orthonormal basis of $\hat{e}_1^{'} = (2\hat{e}_1 + 2\hat{e}_2 + \hat{e}_3)/3$ and $\hat{e}_2^{'} = (\hat{e}_1 - \hat{e}_2)/\sqrt{2}$.

   (a) Express $\hat{e}_3^{'}$ in terms of the old basis $\hat{e}_i$

   (b) If $\overrightarrow{t} = 10\hat{e}_1 + 10\hat{e}_2 - 20\hat{e}_3$, express $\overrightarrow{t}$ in terms of the new basis.

   (c) Show that the Q matrix relating the two bases is orthogonal.

6. Rotated $x_i^{'}$ axes are chosen making angles with the x_j axes as shown in the following table.

-	$x_1^{'}$	$x_2^{'}$	$x_3^{'}$
x_1	90	45	135
x_2	45	60	60
x_3	45	120	120

   (a) Verify that the new axes are a right handed orthogonal system, and display the matrix Q of direction cosines connecting the two coordinate systems.

(b) If tensor $\tilde{T}$ has the components shown in the matrix below, referred to the unprimed coordinate system, compute the matrix of the primed coordinate system components of $\tilde{T}$.

$$\tilde{T} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

7. Find the new components of the tensor provided below if the coordinate system change corresponds to a clockwise rotation of 30 degrees about the x_3 axis.

$$T = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

8. In calculating contact conditions at an interface, it is often necessary to find the unit vector that represents the projection of a given arbitrary vector onto a plane tangent to the interface. If the normal to the tangent plane is denoted as $\hat{n}$, and the arbitrary vector is $\vec{a}$, find $\hat{t}$, the projected unit tangent vector. (Express the result in terms of $\vec{a}$ and $\hat{n}$ and simple vector operations.

9. Consider the following matrix (second order tensor).

$$T = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix}$$

(a) Find the eigenvalues and eigenvectors for the matrix.

(b) Find the transformation matrix, Q, which changes components expressed in the original coordinate system to ones expressed using the eigenvectors as base vectors. Choose the direction associated with the maximum eigenvalue as the new $x_1^{'}$ and the minimum eigenvalue as $x_3^{'}$.

(c) Express matrix T in the new eigenvector coordinate system.